

Fig. 2—Schematic diagram of a microwave coaxial capacitance of the type employed. Only one dc voltage is applied in this device but another terminal is possible.

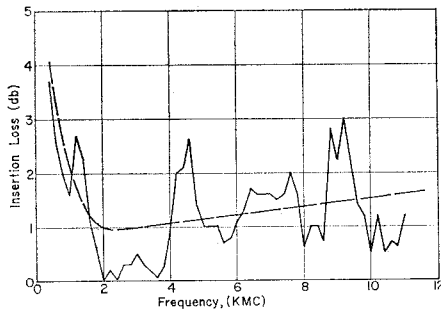


Fig. 3—Experimental measurement of insertion loss of the microwave N-type coaxial capacitance with one dc terminal, showing also the probable value of the connector loss. The connector has a sharp low frequency cutoff and a very gradual high frequency cutoff.<sup>2</sup>

#### RF PROPERTIES

The connector is really a band-pass filter<sup>2</sup> with a sharp low frequency cutoff and a gradual upper frequency cutoff so that it makes an excellent high-pass filter in the range from 1.5 to 10 kmc. A schematic diagram of the equivalent circuit is shown in Fig. 2. In this case  $C_0$  represents the capacitance in the center conductor,  $L$  represents the inductance of the Karma alloy resistance wire,  $C$  represents the capacitance between this wire and the external shield, and  $L_0$  is the natural series inductance between the capacitance  $C_0$  and the dc resistance path. As  $L_0$  approaches zero this becomes a high-pass filter.

The connector loss was measured by the insertion loss method. The power delivered to a matched load from a matched generator was detected with and without the "connector" inserted in the line. The ratio of these powers thus gave the insertion loss directly. As a result of this method the readings were sensitive to the VSWR at the terminals of the connector. The insertion loss varied from 0.01 to 3 db for the frequency range 1.5 to 10 kmc, as shown in Fig. 3.

The dashed average line on Fig. 3 indicates the probable value of the insertion loss across the band in the absence of reflections. Even though the device is a pass band filter the attenuation of the pass band apparently rises very slowly on the high frequency end making the device very broadband.

The VSWR of the connector, which may be of more interest than the insertion loss for some considerations, is shown in Fig. 4.

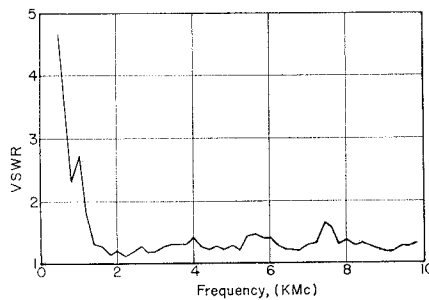


Fig. 4—Experimental measurement of VSWR of the microwave N-type coaxial capacitance with one dc terminal

The VSWR is less than 1.50 between 1.5 and 10 kmc.

No direct measurement was made to determine the radiation loss through the Karma wire terminal, but it was noted that placing a grounded copper shield  $\frac{1}{2}$ -inch long around the terminal caused less than a 2 per cent difference in the VSWR reading when the dc terminal was grounded or open circuited.

C. M. LIN  
R. W. GROW  
Electronics Labs.  
Stanford University  
Stanford, Calif.

### The Cutoff Wavelength of Trough Waveguide\*

#### INTRODUCTION

The trough waveguide,<sup>1</sup> although in use for several years, is not widely known and has received only little attention in the literature. This waveguide was suggested by E. G. Fubini, and its fundamental mode may be compared to that of a TE mode in symmetrical strip transmission line. The configuration is shown in Fig. 1. This type of

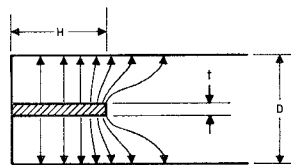


Fig. 1—Trough waveguide.

waveguide has several advantages, which include:

- 1) broad frequency range, because the cutoff frequency of the second mode is approximately three times that of the dominant mode;
- 2) low-reflection, broad-band transitions to TEM lines, easily made by an end-connection of the center conductor

of a coaxial line to a point on the center vane;<sup>2</sup>

- 3) line measurements, made with a minimum of disturbance because of the open side;
- 4) simple control of the propagation characteristics, possible by changing the center vane;
- 5) economical fabrication.

The application of this waveguide to line-source radiators has been investigated extensively by Rotman and Karas and excellent results have been obtained.<sup>3</sup>

A derivation of the cutoff wavelength for a TE mode in symmetrical strip transmission line in the case of a zero-thickness center strip was obtained independently by Jasik<sup>4</sup> and Oliner.<sup>5</sup> In both instances the result was based on the analogous  $E$ -plane bifurcation in rectangular waveguide, and this result applies also to the trough waveguide. In any actual waveguide, however, the center vane must have a finite thickness. It is of interest, therefore, to know how the cutoff wavelength depends on this parameter.

#### DERIVATION OF THE APPROXIMATE CUTOFF WAVELENGTH

By the transverse resonance procedure, the cutoff wavelength is given by

$$\lambda_c = 4(H + d) \quad (1)$$

where  $d$  is the distance from the physical edge of the center vane to the effective open circuit point. In the case of  $t=0$ , we have<sup>4,5</sup>

$$d = \frac{D}{\pi} \ln 2 + \frac{\lambda_c}{2\pi} \left[ S_1 \left( \frac{2D}{\lambda_c} \right) - 2S_1 \left( \frac{D}{\lambda_c} \right) \right], \quad (2)$$

where

$$S_1(x) = \sum_{n=1}^{\infty} \left( \arcsin \frac{x}{n} - \frac{x}{n} \right).$$

We may relate  $d$  to an equivalent fringing capacitance which, because  $d$  is frequency dependent, will also be frequency dependent.

In the case of thick center vane let us consider the equivalent fringing capacitance from one edge of the vane. This is given by

$$C_f = 0.0885\epsilon \frac{d}{\frac{1}{2}(D-t)}, \text{ PF/cm}, \quad (3)$$

so that

$$\frac{d}{D} = \frac{1}{2} \frac{C_f}{0.0885\epsilon} (1 - t/D), \quad (4)$$

and from (1), we have

$$\frac{\lambda_c}{D} = 4 \frac{H}{D} + \frac{2C_f(1 - t/D)}{0.0885\epsilon}. \quad (5)$$

The second term on the right side of (5) is

<sup>2</sup> H. S. Keen, "Scientific Report on Study of Strip Transmission Lines," Airborne Instruments Lab., Mineola, N. Y., Rep. No. 2830-2; December 1, 1955.  
<sup>3</sup> W. Rotman and N. Karas, "Some new microwave antenna designs based on the trough waveguide," 1956 IRE CONVENTION RECORD, pt. 1, pp. 230-235.

<sup>4</sup> H. Jasik, private communication to E. G. Fubini; July 30, 1953.

<sup>5</sup> A. A. Oliner, "Theoretical developments in symmetrical strip transmission line," *Proc. Symp. Modern Advances in Microwave Techniques*, Polytechnic Inst. of Brooklyn, Brooklyn, N. Y., pp. 379-402; November, 1954.

<sup>2</sup> W. P. Mason, "Electro-Mechanical Transducers and Wave Filters," D. Van Nostrand Co., Inc., New York, N. Y., p. 52; 1948.

\* Received by the PGMTT, July 21, 1958.  
<sup>1</sup> Airborne Instruments Lab., Mineola, N. Y., Advertisement, Proc. IRE, vol. 44, p. 2A; August, 1956.

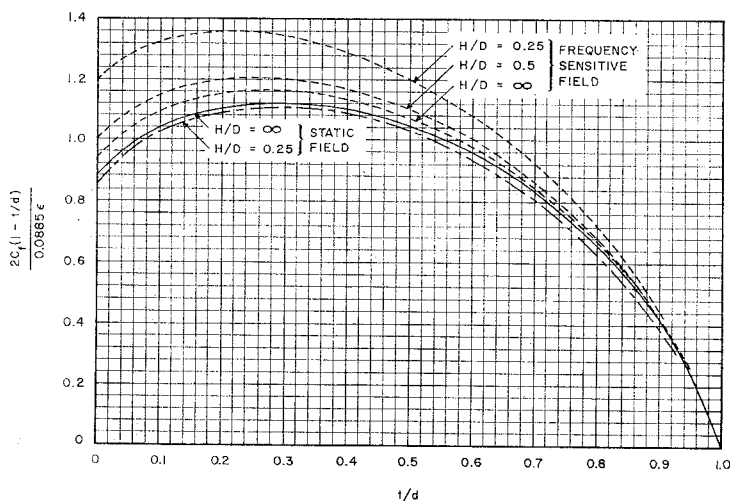


Fig. 2—Normalized fringing capacitance from one edge of trough waveguide center vane.

readily evaluated for *wide* vanes and a *static* field, in which case<sup>6</sup>

$$\frac{2C_f(1-t/D)}{0.0885\epsilon} = \frac{4}{\pi} \ln \left( \frac{2-t/D}{1-t/D} \right) - \frac{2t}{\pi D} \ln \left( \frac{1-[1-t/D]^2}{[1-t/D]^2} \right). \quad (6)$$

This term is plotted in Fig. 2 (solid curve) and for most applications this curve together with (5) will provide sufficiently accurate results.<sup>7</sup>

#### CORRECTIONS TO THE APPROXIMATE SOLUTION

One correction which must be made to the solution obtained above is that due to the assumption of the semi-infinite width of the center vane used in deriving (6). The exact value of the static fringing capacitance may be found, but not in closed, explicit form. Consider the strip transmission line of width  $2H$ , thickness  $t$ , and ground-plane spacing  $D$ , then the capacitance per unit length is given by

$$C = 4(C_b + C_f), \quad (7)$$

where

$$C_b = \frac{0.0885\epsilon H}{\frac{1}{2}(D-t)}$$

is one-half the parallel plate capacitance to

one ground plane, neglecting fringing, and  $C_f$  is the fringing capacitance from one corner of the center strip. Thus

$$\frac{2C_f(1-t/D)}{0.0885\epsilon} = \frac{C(1-t/D)}{0.177\epsilon} - \frac{4H}{D}. \quad (8)$$

Now  $C$  may be evaluated<sup>8</sup> from the relations

$$C = \frac{100\sqrt{\epsilon}}{3Z_0} = 1.1 \pi \sqrt{\epsilon} K'(k)/K(k)$$

$$2H = \frac{2K(k)}{\pi} \left[ \frac{k^2 \operatorname{sn}(u) \operatorname{cn}(u)}{\operatorname{dn}(u)} - \operatorname{zn}(u) \right] \quad (9)$$

$$t/D = \frac{u}{K(k)} - \frac{2K'(k)}{\pi} \left[ \frac{k^2 \operatorname{sn}(u) \operatorname{cn}(u)}{\operatorname{dn}(u)} - \operatorname{zn}(u) \right]$$

where

$K(k)$  is the real quarter period of  $\operatorname{sn}(u)$ ,  $K'(k)$  is the imaginary half period of  $\operatorname{sn}(u)$ ,  $\operatorname{sn}(u)$ ,  $\operatorname{cn}(u)$  and  $\operatorname{dn}(u)$  are Jacobian elliptic functions,  $\operatorname{zn}(u)$  is the Jacobian zeta function.

This evaluation is very tedious, but fortunately the resulting correction is small for all practical cases. The dot-dash curve in Fig. 2 shows the result for the case  $H/D = 0.25$  and, since the deviation from the solid curve ( $H/D$  large) will be less for larger values of  $H/D$ , we may conclude that this correction term is negligible for all practical cases.

<sup>8</sup> R. H. T. Bates, "The characteristic impedance of the shielded slab line," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-4, pp. 28-33; January, 1956.

A correction which is of greater significance is that required to account for the frequency sensitive portion of the fringing capacitance. Neglect of this term in the case  $t=0$  results in an error of less than 4 per cent in the cutoff wavelength for  $H/D$  greater than 0.5, and the error vanishes as  $H/D$  approaches infinity. It does not seem reasonable to expect this term to be strongly dependent on  $t/D$ . It certainly would not have a pole at  $t/D=1$  and, therefore, the second term on the right hand side of (5) will vanish as  $t/D$  approaches unity. An estimate of the effect of frequency sensitivity of the fringing capacitance can be obtained by assuming that it does not depend on  $t/D$ . With this assumption the dashed curves in Fig. 2 are obtained.

From these considerations it is seen that in practical cases, such as a thin, wide center vane, or a thick, narrow center vane, the approximate solution is satisfactory.

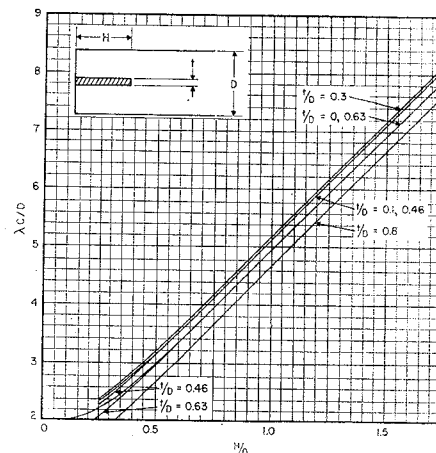


Fig. 3—Cutoff wavelength of trough waveguide.

For convenience, the value of the cutoff wavelength is plotted in Fig. 3 as a function of the waveguide dimensions. It should be pointed out that the curves for  $t/D=0.1$  and  $0.46$  are identical as are those for  $t/D=0.0$  and  $0.63$ . The estimated frequency-dependent correction has been used in this calculation. At  $\lambda_c/D=2$  the higher mode spectrum becomes continuous and a TM mode may also exist.<sup>5</sup> The curves may be extrapolated to larger values of  $H/D$ , but are in doubt in the region  $H/D < 0.5$ . Curves for other values of  $t/D$  are readily obtained from Fig. 2, as is the sensitivity of waveguide wavelength to changes in vane thickness.

K. S. PACKARD  
Airborne Instruments Lab.  
Mineola, N. Y.

<sup>6</sup> J. J. Thomson, "Recent Researches in Electricity and Magnetism," Clarendon Press, Oxford, Eng.; 1893.

<sup>7</sup> This result has been obtained independently by A. A. Oliner, "Leaky Waves on Asymmetric Trough Waveguides," Microwave Res. Inst., Polytechnic Inst. of Brooklyn, Brooklyn, N. Y., Memo. No. 36; October 31, 1957.

